Math 2263

Quiz 12

Name Section Score

(8 points) Let $F(x,y) = \frac{-y\mathbf{i}+x\mathbf{j}}{x^2+y^2}$,

(a) Show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

(b) Show that $\int\limits_C \mathbf{F} \cdot \boldsymbol{dr}$ is NOT independent of path. Does this contradict Theorem 6? [Hint: Compute $\int\limits_{C_1} \mathbf{F} \cdot \boldsymbol{dr}$ and $\int\limits_{C_2} \mathbf{F} \cdot \boldsymbol{dr}$, where C_1 and C_2 are the upper and lower halves of the circle $x^2 + y^2 = 1$ from (1, 0) to (-1, 0)].

Theorem 6: Let $F = P\mathbf{i} + Q\mathbf{j}$ be a vector field on an open simply-connected region D. Suppose P and Q have continuous first-order derivatives and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D, then F is conservative.

(7 points) Let D be a region bounded by a simple closed path C in the xy-plane. Use Green's Theorem to prove that the coordinates of the centroid $(\bar{x}, \bar{y})$ of D are

$$\bar{x} = \frac{1}{2A} \oint\limits_C x^2 dy, \ \bar{y} = -\frac{1}{2A} \oint\limits_C y^2 dx$$

where A is the area of D.